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8-9 Study Guide and Intervention

Perfect Squares

Factor Perfect Square Trinomials

Perfect Square Trinomial	a trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

The patterns shown below can be used to factor perfect square trinomials.

Squaring a Binomial	Factoring a Perfect Square Trinomial
$(a + 4)^2 = a^2 + 2(a)(4) + 4^2$ $= a^2 + 8a + 16$	$a^2 + 8a + 16 = a^2 + 2(a)(4) + 4^2$ $= (a + 4)^2$
$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + 3^2$ $= 4x^2 - 12x + 9$	$4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$ $= (2x - 3)^2$

Example 1 **Determine whether $16n^2 - 24n + 9$ is a perfect square trinomial. If so, factor it.**

Since $16n^2 = (4n)(4n)$, the first term is a perfect square.

Since $9 = 3 \cdot 3$, the last term is a perfect square.

The middle term is equal to $2(4n)(3)$.

Therefore, $16n^2 - 24n + 9$ is a perfect square trinomial.

$16n^2 - 24n + 9 = (4n)^2 - 2(4n)(3) + 3^2$
$= (4n - 3)^2$

Example 2 **Factor $16x^2 - 32x + 15$.**

Since 15 is not a perfect square, use a different factoring pattern.

$16x^2 - 32x + 15$ Original trinomial
$= 16x^2 + mx + px + 15$ Write the pattern.
$= 16x^2 - 12x - 20x + 15$ $m = -12$ and $p = -20$
$= (16x^2 - 12x) - (20x - 15)$ Group terms.
$= 4x(4x - 3) - 5(4x - 3)$ Find the GCF.
$= (4x - 5)(4x - 3)$ Factor by grouping.

Therefore $16x^2 - 32x + 15 = (4x - 5)(4x - 3)$.

Exercises

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. $x^2 - 16x + 64$
yes; $(x - 8)(x - 8)$

2. $m^2 + 10m + 25$
yes; $(m + 5)(m + 5)$

3. $p^2 + 8p + 64$
no

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

4. $98x^2 - 200y^2$
$2(7x + 10y)(7x - 10y)$

5. $x^2 + 22x + 121$
$(x + 11)^2$

6. $81 + 18j + j^2$
$(9 + j)^2$

7. $25c^2 - 10c - 1$
prime

8. $169 - 26r + r^2$
$(13 - r)^2$

9. $7x^2 - 9x + 2$
$(7x - 2)(x - 1)$

10. $16m^2 + 48m + 36$
$4(2m + 3)^2$

11. $16 - 25a^2$
$(4 + 5a)(4 - 5a)$

12. $b^2 - 16b + 256$
prime

13. $36x^2 - 12x + 1$
$(6x - 1)^2$

14. $16a^2 - 40ab + 25b^2$
$(4a - 5b)^2$

15. $8m^3 - 64m$
$8m(m^2 - 8)$

Lesson 8-9

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8-9 Study Guide and Intervention *(continued)*

Perfect Squares

Solve Equations with Perfect Squares Factoring and the Zero Product Property can be used to solve equations that involve repeated factors. The repeated factor gives just one solution to the equation. You may also be able to use the **Square Root Property** below to solve certain equations.

Square Root Property	For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example **Solve each equation. Check your solutions.**

a. $x^2 - 6x + 9 = 0$

$x^2 - 6x + 9 = 0$	Original equation
$x^2 - 2(3x) + 3^2 = 0$	Recognize a perfect square trinomial.
$(x - 3)(x - 3) = 0$	Factor the perfect square trinomial.
$x - 3 = 0$	Set repeated factor equal to 0.
$x = 3$	Solve.

The solution set is {3}. Since $3^2 - 6(3) + 9 = 0$, the solution checks.

b. $(a - 5)^2 = 64$

$(a - 5)^2 = 64$	Original equation
$a - 5 = \pm\sqrt{64}$	Square Root Property
$a - 5 = \pm 8$	$64 = 8 \cdot 8$
$a = 5 \pm 8$	Add 5 to each side.
$a = 5 + 8$ or $a = 5 - 8$	Separate into 2 equations.
$a = 13$ $\quad a = -3$	Solve each equation.

The solution set is {−3, 13}. Since $(-3 - 5)^2 = 64$ and $(13 - 5)^2 = 64$, the solutions check.

Exercises

Solve each equation. Check the solutions.

1. $x^2 + 4x + 4 = 0$ **{−2}**

2. $16n^2 + 16n + 4 = 0$ $\left\{-\frac{1}{2}\right\}$

3. $25d^2 - 10d + 1 = 0$ $\left\{\frac{1}{5}\right\}$

4. $x^2 + 10x + 25 = 0$ **{−5}**

5. $9x^2 - 6x + 1 = 0$ $\left\{\frac{1}{3}\right\}$

6. $x^2 + x + \frac{1}{4} = 0$ $\left\{-\frac{1}{2}\right\}$

7. $25k^2 + 20k + 4 = 0$ $\left\{-\frac{2}{5}\right\}$

8. $p^2 + 2p + 1 = 49$ **{−8, 6}**

9. $x^2 + 4x + 4 = 64$ **{−10, 6}**

10. $x^2 - 6x + 9 = 25$ **{−2, 8}**

11. $a^2 + 8a + 16 = 1$ **{−3, −5}**

12. $16y^2 + 8y + 1 = 0$ $\left\{-\frac{1}{4}\right\}$

13. $(x + 3)^2 = 49$ **{−10, 4}**

14. $(y + 6)^2 = 1$ **{−7, −5}**

15. $(m - 7)^2 = 49$ **{0, 14}**

16. $(2x + 1)^2 = 1$ **{−1, 0}**

17. $(4x + 3)^2 = 25$ $\left\{-2, \frac{1}{2}\right\}$

18. $(3h - 2)^2 = 4$ $\left\{\frac{4}{3}, 0\right\}$

19. $(x + 1)^2 = 7$ $\{-1 \pm \sqrt{7}\}$

20. $(y - 3)^2 = 6$ $\{3 \pm \sqrt{6}\}$

21. $(m - 2)^2 = 5$ $\{2 \pm \sqrt{5}\}$