

**10-4 Study Guide and Intervention****Radical Equations**

**Radical Equations** Equations containing radicals with variables in the radicand are called **radical equations**. These can be solved by first using the following steps.

**Step 1** Isolate the radical on one side of the equation.

**Step 2** Square each side of the equation to eliminate the radical.

**Example 1** Solve  $16 = \frac{\sqrt{x}}{2}$  for  $x$ .

$$16 = \frac{\sqrt{x}}{2} \quad \text{Original equation}$$

$$2(16) = 2\left(\frac{\sqrt{x}}{2}\right) \quad \text{Multiply each side by 2.}$$

$$32 = \sqrt{x} \quad \text{Simplify.}$$

$$(32)^2 = (\sqrt{x})^2 \quad \text{Square each side.}$$

$$1024 = x \quad \text{Simplify.}$$

The solution is 1024, which checks in the original equation.

**Example 2** Solve  $\sqrt{4x - 7} + 2 = 7$ .

$$\sqrt{4x - 7} + 2 = 7 \quad \text{Original equation}$$

$$\sqrt{4x - 7} + 2 - 2 = 7 - 2 \quad \text{Subtract 2 from each side.}$$

$$\sqrt{4x - 7} = 5 \quad \text{Simplify.}$$

$$(\sqrt{4x - 7})^2 = 5^2 \quad \text{Square each side.}$$

$$4x - 7 = 25 \quad \text{Simplify.}$$

$$4x - 7 + 7 = 25 + 7 \quad \text{Add 7 to each side.}$$

$$4x = 32 \quad \text{Simplify.}$$

$$x = 8 \quad \text{Divide each side by 4.}$$

The solution is 8, which checks in the original equation.

**Exercises**

Solve each equation. Check your solution.

1.  $\sqrt{a} = 8$  **64**

2.  $\sqrt{a} + 6 = 32$  **676**

3.  $2\sqrt{x} = 8$  **16**

4.  $7 = \sqrt{26 - n}$  **-23**

5.  $\sqrt{-a} = 6$  **-36**

6.  $\sqrt{3r^2} = 3$   **$\pm\sqrt{3}$**

7.  $2\sqrt{3} = \sqrt{y}$  **12**

8.  $2\sqrt{3a} - 2 = 7$   **$6\frac{3}{4}$**

9.  $\sqrt{x - 4} = 6$  **40**

10.  $\sqrt{2m + 3} = 5$  **11**

11.  $\sqrt{3b - 2} + 19 = 24$  **9**

12.  $\sqrt{4x - 1} = 3$   **$\frac{5}{2}$**

13.  $\sqrt{3r + 2} = 2\sqrt{3}$   **$\frac{10}{3}$**

14.  $\sqrt{\frac{x}{2}} = \frac{1}{2}$   **$\frac{1}{2}$**

15.  $\sqrt{\frac{x}{8}} = 4$  **128**

16.  $\sqrt{6x^2 + 5x} = 2$   **$\frac{1}{2}, -\frac{4}{3}$**

17.  $\sqrt{\frac{x}{3}} + 6 = 8$  **12**

18.  $2\sqrt{\frac{3x}{5}} + 3 = 11$   **$26\frac{2}{3}$**

# 10-4 Study Guide and Intervention *(continued)*

## Radical Equations

**Extraneous Solutions** To solve a radical equation with a variable on both sides, you need to square each side of the equation. Squaring each side of an equation sometimes produces **extraneous solutions**, or solutions that are not solutions of the original equation. Therefore, it is very important that you check each solution.

**Example 1** Solve  $\sqrt{x + 3} = x - 3$ .

|                                |                                    |
|--------------------------------|------------------------------------|
| $\sqrt{x + 3} = x - 3$         | Original equation                  |
| $(\sqrt{x + 3})^2 = (x - 3)^2$ | Square each side.                  |
| $x + 3 = x^2 - 6x + 9$         | Simplify.                          |
| $0 = x^2 - 7x + 6$             | Subtract $x$ and 3 from each side. |
| $0 = (x - 1)(x - 6)$           | Factor.                            |
| $x - 1 = 0$ or $x - 6 = 0$     | Zero Product Property              |
| $x = 1$ $x = 6$                | Solve.                             |

|                                      |                                      |
|--------------------------------------|--------------------------------------|
| <b>CHECK</b> $\sqrt{x + 3} = x - 3$  | $\sqrt{x + 3} = x - 3$               |
| $\sqrt{1 + 3} \stackrel{?}{=} 1 - 3$ | $\sqrt{6 + 3} \stackrel{?}{=} 6 - 3$ |
| $\sqrt{4} \stackrel{?}{=} -2$        | $\sqrt{9} \stackrel{?}{=} 3$         |
| $2 \neq -2$                          | $3 = 3 \checkmark$                   |

Since  $x = 1$  does not satisfy the original equation,  $x = 6$  is the only solution.

### Exercises

Solve each equation. Check your solution.

1.  $\sqrt{a} = a$  **0, 1**

2.  $\sqrt{a + 6} = a$  **3**

3.  $2\sqrt{x} = x$  **0, 4**

4.  $n = \sqrt{2 - n}$  **1**

5.  $\sqrt{-a} = a$  **0**

6.  $\sqrt{10 - 6k} + 3 = k$   **$\emptyset$**

7.  $\sqrt{y - 1} = y - 1$  **1, 2**

8.  $\sqrt{3a - 2} = a$  **1, 2**

9.  $\sqrt{x + 2} = x$  **2**

10.  $\sqrt{2b + 5} = b - 5$  **10**

11.  $\sqrt{3b + 6} = b + 2$  **-2, 1**

12.  $\sqrt{4x - 4} = x$  **2**

13.  $r + \sqrt{2 - r} = 2$  **1, 2**

14.  $\sqrt{x^2 + 10x} = x + 4$  **8**

15.  $-2\sqrt{\frac{x}{8}} = 15$   **$\emptyset$**

16.  $\sqrt{6x^2 - 4x} = x + 2$

17.  $\sqrt{2y^2 - 64} = y$

18.  $\sqrt{3x^2 + 12x + 1} = x + 5$

$-\frac{2}{5}, 2$

**8**

**-4, 3**